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Numerical analyses of the collision of localized structures in the Davey-Stewartson equations

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Collisions of two localized structures called "dromions," whose time evolutions are described by the Davey-Stewartson equations, are investigated numerically for various situations. It is observed generally that the initial dromions break into four pulses after collisions. The intermediate states of the collision show interesting behaviors: a new pulse is created and it oscillates during the collision. It is found that the size of each of the four pulses in the final stage is determined by a balance of the period of the oscillation and the relative velocity of the initial two dromions. When the masses of the initial dromions are different, pulses are subject to undergo exchange of the masses due to collision. These behaviors are expected to be observed in a real system, such as fluid dynamics and plasma physics.

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I. INTRODUCTION

Localized structures in multidimensions are one of the recent interests for researchers in various fields. They are worth studying from both a theoretical and a practical point of view. Dromion [1] is an example of such a structure that appears in the system described by the Davey-Stewartson 1 (DS1) equations [2]

$$iA_t + A_{xx} + A_{yy} - 2|A|^2A + (Q_x + Q_y)A = 0, \qquad (1)$$

$$Q_{xy} = |A|_x^2 + |A|_y^2. \qquad (2)$$

One of the characteristics of the dromion solutions to emphasize is that the main flow A is localized in two-dimensional space, while the mean flow Q is not. The mean flow is driven at the boundaries like a one-dimensional soliton [3] and plays an important role in conveying the localized structures of the main flow [3,4]. The DS1 equations are derived in many branches of physics, such as fluid dynamics [5] or plasma physics [6]. It has been shown that an electrostatic ion wave propagating perpendicularly to an applied magnetic field is well described by the DS1 equations [6]. Then there is the possibility that the electrostatic potential of the ion wave will localize two dimensionally like a "bell shape" and the mean current of the ion will have an effect on the mean flow.

The stability of dromions can ensure us the observation of localized structures in real multidimensional systems. In our previous paper [7] we analyzed numerically the time evolutions of the dromion solutions. We showed that a single dromion propagates stably in the Lyapunov sense and that in the special case of a collision of two dromions, the initial dromions break into four stable pulses after the collision.

This paper is devoted to the detailed numerical analysis of the stability of dromions against their collisions. We take various superpositions of the two one-dromion solutions as initial conditions. There are three main parameters to determine which characterize the collision: a relative velocity, a mass ratio of the initial two dromions, and an impact parameter. We aim to clarify the mechanism of the appearance of four stable pulses due to the collision by changing these parameters.

There are some exact solutions that show the creation or the annihilation of pulses and these exhibit configurations close to our results. It should be noted, however, that our cases are different from these exact dromion solutions since the boundary conditions of the mean flows in our simulations are different from the exact ones [7]. We shall examine whether or not the result of the behavior in the simulations can be explained by using the exact solutions.

It is significant to carry out numerical analyses for the following reasons. First, little is known about the behavior of the solutions that deviate from exact ones such as that two one-dromion collisions. Second, the Lyapunov analysis [8] cannot be used to investigate the stability of the dromion solutions. This analysis is applicable only when the equation under consideration has a Hamiltonian. In the case of the nonlinear Schrödinger equation, the stability of localized pulses is studied in detail by this method. On the contrary, for the dromion solution of the DS1 equations, we emphasize that the Hamiltonian cannot be constructed (see the Appendix).

The outline of this paper is as follows. In Sec. II, we briefly explain the numerical method and boundary conditions of the simulation. In Sec. III, numerical results

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of dromion collisions in various cases are presented. We compare our results with some of the exact dromion solutions in Sec. IV, and give concluding remarks in Sec. V.

II. THE ONE-DROMION SOLUTION AND THE NUMERICAL METHOD

A. The one-dromion solution

Let us summarize the one-dromion solution of the DS1 equations. Equations (1) and (2) can be rewritten in another form:

$$iA_t + A_{xx} + A_{yy} + (U+V)A = 0, \tag{3}$$

$$U_y = (|A|^2)_x, \qquad V_x = (|A|^2)_y, \tag{4}$$

where $U = Q_x - |A|^2$ and $V = Q_y - |A|^2$. The one-dromion solution is given by [4]

$$A = \frac{G}{F}. \tag{5}$$

Here we have chosen

$$\begin{aligned} F &= 1 + \exp(\eta_1 + \eta_1^*) + \exp(\eta_2 + \eta_2^*) \\ &\quad + \gamma \exp(\eta_1 + \eta_1^* + \eta_2 + \eta_2^*), \\ G &= \rho \exp(\eta_1 + \eta_2). \end{aligned} \tag{6}$$

The parameters in (6) are given as

$$\begin{aligned} |\rho| &= 2\sqrt{2k_r l_r(\gamma - 1)}, \\ \eta_1 &= (k_r + ik_i)x + (\Omega_r + i\Omega_i)t, \\ \eta_2 &= (l_r + il_i)y + (\omega_r + i\omega_i)t, \\ \Omega_r &= -2k_r k_i, \\ \omega_r &= -2l_r l_i, \qquad \omega_i + \Omega_i = k_r^2 + l_r^2 - k_i^2 - l_i^2. \end{aligned}$$

In this paper, we take Ω_i as 1/2. The constants γ, k_r, k_i, l_r, and l_i are real, which are substantial free parameters in the one-dromion solution. The constant γ determines an amplitude, k_r the width of the pulse in the x direction, and l_r that in the y direction. The quantities k_i and l_i are x and y components of velocity, respectively.

We obtain the potentials U and V, which are determined by integrating (4):

$$U = 2(\ln F)_{xx}, \qquad V = 2(\ln F)_{yy}. \tag{7}$$

From (7) we can get the cross section of U and V at the boundary $y, x = -\infty$, respectively:

$$U|_{y=-\infty} = \frac{8k_r^2 \exp(\eta_1 + \eta_1^*)}{[1 + \exp(\eta_1 + \eta_1^*)]^2}, \tag{8}$$

$$V|_{x=-\infty} = \frac{8l_r^2 \exp(\eta_2 + \eta_2^*)}{[1 + \exp(\eta_2 + \eta_2^*)]^2}. \tag{9}$$

This is crucial for the dromion solution driving the potentials from the boundaries as (8) and (9).

B. Numerical method

Next, we describe the numerical method briefly. We carry out the computation in a region $[-p,p]\times[-p,p]$, which means the area $|x| \leq p$, $|y| \leq p$ in the xy plane. This area is transformed into $[0,2\pi]\times[0,2\pi]$ by the transformations of variables $x \to \pi(x+p)/p$ and $y \to \pi(y+p)/p$. We take p as 15 and the grid as 64×64 throughout simulations. The space derivative in (3) is performed by using the psudospectral method [9] with periodic boundary conditions. Time integration is performed by both the Burilsh and Store method [10] and the fourth-order Runge-Kutta method with the appropriate accuracy of adaptive step size control. Equation (4) is calculated by the fourth-order Runge-Kutta method with boundary conditions (8) and (9). We use a cubic spline when the midpoint value between meshes is needed. We evaluate the first conserved quantity $I_1 = \int |A|^2 dv$ with appropriate accuracy.

Finally, let us examine the accuracy of using the boundary conditions (8) and (9) in these simulations. In an exact sense, because the simulations have been performed in the region $[-p,p]\times[-p,p]$, we must choose the value of $U(x, y=-p)$ as the boundary condition of U and $V(x=-p, y)$ as that of V, respectively. The function of (8) and (9), however, is useful in a practical sense because of its simplicity. Of course the two conditions are identical if we could take $p = \infty$. We compared these two boundary conditions and the differences between them are listed in Table I. The two performances correspond with remarkably high accuracy. Hence, hereafter in this paper, we choose the cross sections (8) and (9).

III. RESULTS OF SIMULATIONS

In this section we present the numerical results of the single-dromion collisions. We take superpositions of the two one-dromion solutions as the initial conditions. It is obvious that these cannot be exact solutions of the nonlinear equations since we take only superpositions of a single-dromion solution. In Sec. IV we shall discuss in detail the difference between the exact dromion solution and ours.

We begin the study by changing the relative velocity and keeping the mass ratio as 1 and the impact parameter as zero (i.e., head-on collisions of two identical dromions in various relative velocities). We take $k1_r = k2_r = l1_r = l2_r = 0.8$ and $\gamma 1 = \gamma 2 = 3.0$ and choose the following cases for the velocities:

$$(k1_i, k2_i, l1_i, l2_i) = (W/8,\ -W/8,\ W/8,\ -W/8), \qquad W = 3, 4, 5, 6.$$

We see that the constant of motion $I_1 = \int |A|^2 dv$ has been conserved with high accuracy in all of the computations reported here (the maximum fluctuation of I_1 during calculation is $\Delta I_1/I_1 \sim 10^{-15}$).

The collision for $W = 5$ is shown in Fig. 1. This figure shows typical aspects of the collision. As two dromions approach each other, both of them emit their parts and

TABLE I. Time variations of maximum amplitude of $|A|^2$ in the two cases of boundaries with parameters $\gamma = 3$, $k_r = l_r = 4/5$, and $k_i = l_i = 4/8$. A single dromion propagates stably in the Lyapunov sense and there are little differences between them.

Time	Exact boundary conditions	Infinite boundary conditions	10^{10}(Difference)
−3.000000	0.3408389281735529	0.3408389281735529	0.0
−1.355706	0.3417900100132564	0.3417900099430654	2.053628
−0.794856	0.3377552920420104	0.3377552918767084	4.894135
0.307718	0.3372104103726524	0.3372104103653520	0.2164950
1.007018	0.3428532835640901	0.3428532834641558	2.914784
1.322360	0.3421235844780174	0.3421235843649262	3.305566
1.900000	0.3421816909319298	0.3421816908061519	3.675763
2.408726	0.3384982192722150	0.3384982190838372	5.565105
2.867634	0.3378210317208404	0.3378210316233613	2.885524

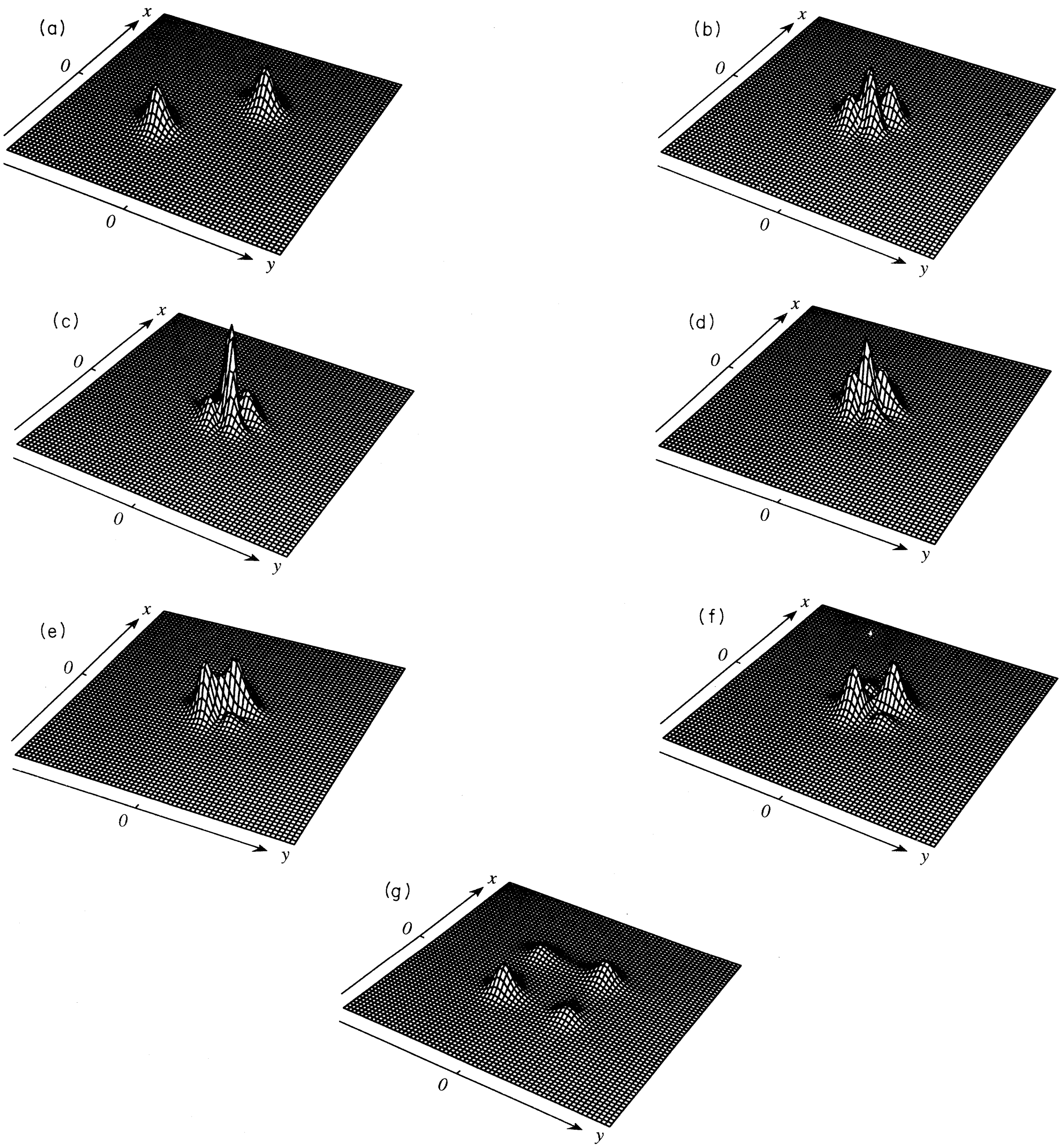

FIG. 1. Solid profiles of $|A|^2$ in the case of the collision with parameters $\gamma 1 = \gamma 2 = 3$, $k1_r = k2_r = l1_r = l2_r = 4/5$, $k1_i = l1_i = 5/8$, and $k2_i = l2_i = -5/8$ at (a) $t = -3.0$, (b) $t = -0.76$, (c) $t = 0.0$, (d) $t = 0.23$, (e) $t = 0.5$, (f) $t = 1.5$, and (g) $t = 3.5$. After the collision, four pulses appear that propagate almost stably.

a third pulse is formed midway between them. The pulse becomes higher and flat on $x = -y$, while the two dromions become smaller [Figs. 1(b) and 1(c)]. These dromions remain as small side pulses. In the next stage, the side pulses approach each other and become larger, again approaching the midway pulse [Fig. 1(d)]. Then, the midway pulse becomes smaller and the two pulses on $x = y$ larger. We can observe two small lumps on $x = -y$, which are the remainders of the edges of the midway pulse [Fig. 1(e)]. Finally, the two lumps become larger and four pulses appear. These four pulses gradually separate and propagate almost stably [Fig. 1(f) and 1(g)]. There are also small ripples of the main flow along potentials U and V. The amplitudes of the ripples are less than 8% of the heights of the pulses. We find that the two pulses on $x = -y$ are similar. It should be noted that the aspects of the collision are highly symmetrical with respect to $x = y$ throughout the calculations. Theoretically speaking, the system has this symmetry and the results of the simulation precisely agree with this. On the other hand, the two pulses on $x = y$ in the final stage do not look similar. We think that this is because of the following reasons. First, the system is asymmetrical, but not so much with respects to $x = -y$ (see the Appendix). Second, ripples generated by the collision may affect asymmetrically the two pulses on $x = y$. This seems to be the more dominant reason. We believe that if we could realize an ideal condition, so that the ripples are small enough, the two pulses on $x = y$ would be similar.

The reason that four pulses appear in the final stage can be understood as follows. Figures 2–5 are the contour plots of the quantities A, U, and V during collisions with different relative velocities. It is observed that the four pulses are located around the cross points of U and V, where the peaks of the potentials overlap. Considering (3) and (4), we easily see that the potentials U and V are attractive in this case. Then the cross points of them are the most attractive points in the entire region. This explains that the cross points of potentials attract main flow A and it is natural that four pulses appear after the collision. As we have mentioned above, the ripples are also attracted to them. From this we find that the position of the four cross points plays an important role in the collision of dromions.

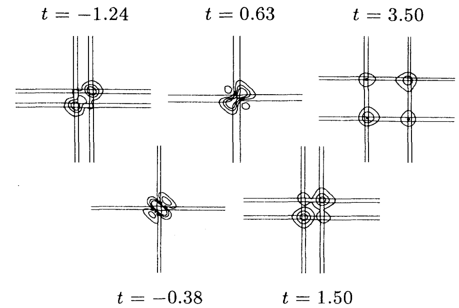

FIG. 3. Contours of the collision of identical dromions with velocities $(k1_i, k2_i, l1_i, l2_i) = (5/8,\ -5/8,\ 5/8,\ -5/8)$. The heights of the contours are chosen to be the same values as those in Fig. 2.

It is interesting to note here that the two pulses on $x = y$ are larger than those on $x = -y$ in the cases of Figs. 2 and 3, although in the cases of Figs. 4 and 5 the two pulses on $x = -y$ are larger than those on $x = y$. Let us explain this phenomenon.

From these figures, especially from Fig. 5, the large pulse midway on the origin oscillates between two states: a flat pulse in the direction along $x = y$ and that along $x = -y$. This oscillation occurs during the period that the two pulses of U and those of V are sufficiently close, respectively. We can evaluate the periods of the oscillations from these figures. The periods do not change remarkably in all of these cases and half of a period $T/2$=1–1.5. Thus we think that the period may be irrelevant to the relative velocity. In order to corroborate this assertion, we simulate the collision with the velocities $(k1_i, k2_i, l1_i, l2_i)=(1/8, -1/8, 1/8, -1/8)$ (Fig. 6). We observe that the midway pulse oscillates during four periods of it and confirm the universal nature of the oscillation that the half period of each oscillation is 1–1.5.

This oscillation plays an important role in the final stages of collisions. Before the collision, we observe four cross points of the mean flow, which play the role of attracting points. As these four points approach, a strong

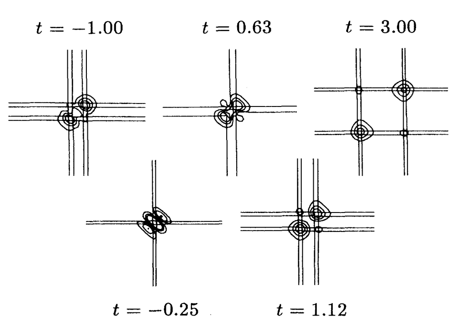

FIG. 2. Contours of the collision of identical dromions, which has the same size as in Fig. 1, with velocities $(k1_i, k2_i, l1_i, l2_i) = (6/8,\ -6/8,\ 6/8,\ -6/8)$. The heights of the contours are 0.03, 0.1, and 0.2 for $|A|^2$ and 90% of the maximum values for U and V.

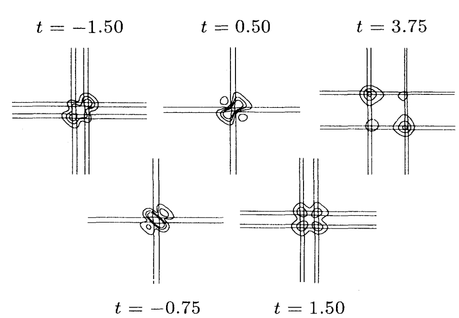

FIG. 4. Contours of the collision of identical dromions with velocities $(k1_i, k2_i, l1_i, l2_i) = (4/8,\ -4/8,\ 4/8,\ -4/8)$. The heights of the contours are chosen to be the same values as those in Fig. 2.

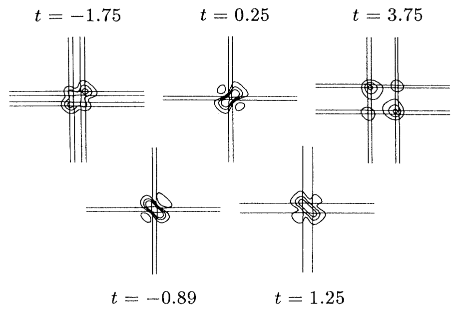

FIG. 5. Contours of the collision of similar dromions with velocities $(k1_i, k2_i, l1_i, l2_i) = (3/8,\ -3/8,\ 3/8,\ -3/8)$. The heights of the contours are chosen to be the same values as those in Fig. 2.

attractive area is formed around the origin. Then, since structures of the main flow are attracted to this point, some parts of the dromions flow into this area to form the midway pulse. The new pulse is strongly bounded to the attractive area and hence it oscillates because of the inertia of the fluid. When the pulses of the mean flows sufficiently separate away in the course of time, the attracting area at the origin will be divided again into four areas around each of the intersections of the mean flows. The oscillation stops at this stage and the midway pulse is separated into four pulses, each of which is located at the intersection of the mean flows. Therefore, the timing of the separation of the potentials, which is related to the relative velocity, and the oscillations of the midway pulse determine how the initial two dromions break into four pulses in the final stage of their collision [7]. We call this mechanism the "distribution law" in this paper. In Figs. 2 and 3 the potentials separate when the midway pulse is flat along $x = y$; then, as a result, the two pulses on $x = y$ are larger than the other two in their final stages. The situations in Figs. 4 and 5 are opposite those in Figs. 2 and 3. Thus we have explained the differences in the final stages.

Next, we carry out numerical simulations under other conditions, by changing the impact parameter slightly

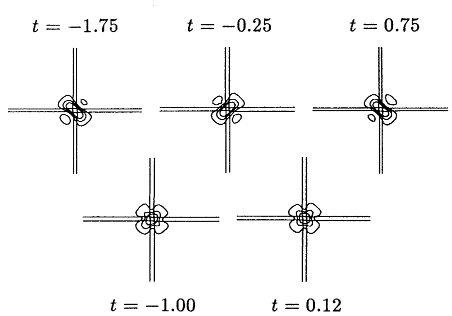

FIG. 6. Contours of a period of the intermediate oscillations in the collision with velocities $(k1_i, k2_i, l1_i, l2_i) = (1/8,\ -1/8,\ 1/8,\ -1/8)$. The heights of the contours are chosen to be the same values as those in Fig. 2. The oscillation occurs for four periods in this case and half of a period is observed to be about $T/2 = 1$–1.5.

from zero and the mass ratio slightly from unity. Figure 7 shows the typical circumstance of the collision with a finite impact parameter, which is small compared to the widths of the pulses of the mean flows. Observing almost the same phenomena as before, we can conclude that a small impact parameter does not affect the aspects of the collision.

In Fig. 8 we change the mass ratio slightly from unity. Also in this case an oscillating pulse is formed when the two dromions collide and four pulses appear after the collision. However, there is a difference from the previous cases in that the larger dromion emits more of itself than the smaller one. Moreover, in the final stage, the larger pulse on the line $x = y$ is in the first quadrant and the smaller one on the same line is in the third, although we have chosen the opposite mass distribution for the initial configuration. This means that the initial dromions exchange their masses with each other through the interaction, thus breaking into four pieces.

IV. COMPARISON WITH EXACT DROMION SOLUTIONS

We investigate the the exact (2,2)-dromion solutions in this section. By (M, N) dromion we mean a solution whose mean flow U has asymptotically M pulses in $y \to -\infty$ and V has N pulses in $x \to -\infty$. Some of the (2,2)-dromion solutions show configurations similar to our numerical results, and we are going to examine whether the distribution law can be explained by them. An exact (2,2)-dromion solution is written in terms of Gramian as [11]

$$F = |I + K\Phi|, \tag{10}$$

where I, K, and Φ are 4×4 matrices. The matrix I is the identity and K is a constant Hermitian matrix whose elements are denoted by $K = (a_{mn})$, $m, n \in \{1, \ldots, 4\}$. The explicit form of Φ is

$$\Phi = \begin{pmatrix} \int_{-\infty}^{x} \phi_i \phi_j^* dx & 0 \\ 0 & \int_{y}^{\infty} \psi_k \psi_l^* dy \end{pmatrix}, \quad i, j \in \{1, 2\}, \quad k, l \in \{1, 2\}. \tag{11}$$

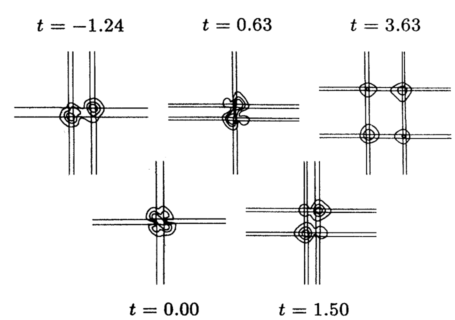

FIG. 7. Contours of the collision with parameters $\gamma 1 = \gamma 2 = 3$, $k1_r = k2_r = l1_r = l2_r = 4/5$, $k1_i = l1_i = 5/8$, and $k2_i = l2_i = -5/8$. The impact parameter is 1.0, which is evaluated in the computation region $[-p, p] \times [-p, p]$.

Here an asterisk denotes a complex conjugate. The functions ϕ_i and ψ_k are

$$\phi_i = \exp(p_i x - i p_i^2 t), \quad \psi_k = \exp(-q_k y + i q_k^2 t), \quad i,k = 1,2.$$

We should take the real parts of the parameters p_i and q_k as positive constants in order to ensure the regularity of the solution. Let us construct an exact solution that has the following property: the initial two dromions on $x = y$ break into four after the collision. We impose the components of K as follows [11].

First, we assume the condition that there are no dromions on $x = -y$ at $t = -\infty$:

$$a_{13} = 0, \quad a_{24} = a_{23}a_{34} + a_{12}a_{14}. \tag{12}$$

Second, we take the condition that the amplitudes of the initial two and the final four dromions are not zero:

$$a_{14}, a_{23}, a_{24} \neq 0, \quad a_{23} \neq a_{24}a_{43}, \quad a_{14} \neq a_{12}a_{24},$$
$$a_{12}(a_{23} - a_{24}a_{43}) + a_{14}(a_{43} - a_{23}a_{42}) \neq 0. \tag{13}$$

There exist nontrivial $a_{m,n}$'s of Eqs. (12) and (13) that generate solutions that are similar to the results of our computations only in the initial and the final stages. However, the intermediate states of these solutions are different in general. Let us consider this in detail.

The intermediate states depend on the behaviors of the mean flows. For example, the boundary condition of V at $x = -\infty$ is given by (7), where F is

$$F = 1 + \frac{1}{q_1 + q_1^*}\psi_1\psi_1^* + \frac{1}{q_2 + q_2^*}\psi_2\psi_2^* + a_{34}\frac{1}{q_1 + q_2^*}\psi_1\psi_2^*$$
$$+ a_{34}^*\frac{1}{q_2 + q_1^*}\psi_2\psi_1^*$$
$$+ (1 - |a_{34}|^2)\frac{|q_1 - q_2|^2}{(q_1 + q_1^*)(q_2 + q_2^*)(q_1 + q_2^*)(q_2 + q_1^*)}$$
$$\times \psi_1\psi_1^*\psi_2\psi_2^*. \tag{14}$$

Then the exact $V|_{x=-\infty}$ derived from (14) is apparently different from our boundary, which is a superposition of $V|_{x=-\infty}$ of two one-dromion solutions. The exact one has

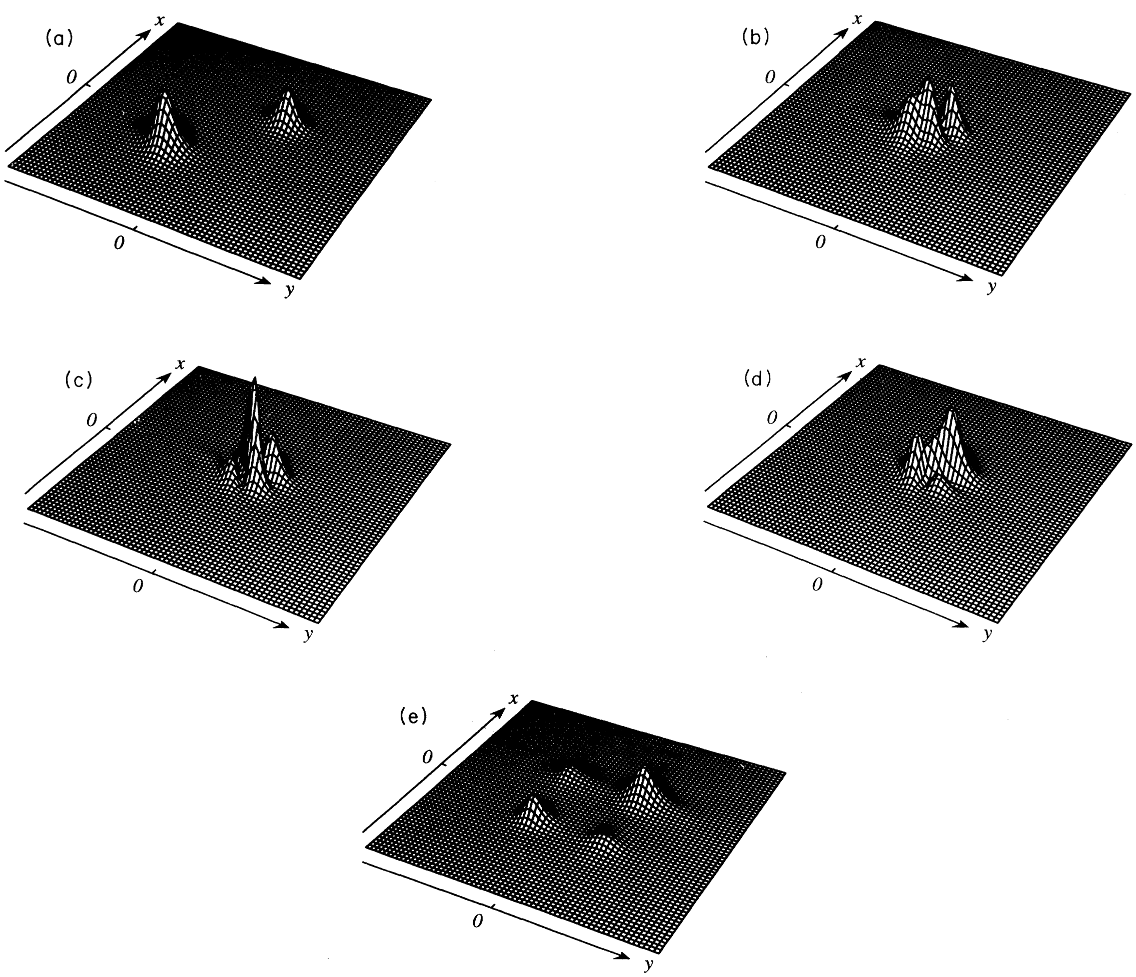

FIG. 8. Solid profiles of $|A|^2$ in the case of a head-on collision with parameters $\gamma 1 = 6, \gamma 2 = 2$, $k1_r = l1_r = 0.7$, $k2_r = l2_r = 0.9$, $k1_i = l1_i = 5/8$, and $k2_i = l2_i = -5/8$ at (a) $t = -3$, (b) $t = -0.74$, (c) $t = 0.0$, (d) $t = 0.5$, and (e) $t = 3.0$. (b) and (c) As two dromions approach, the larger dromion emits its part more than the smaller one. In the final stage, the larger and the smaller pulses on $x = y$ appear in the position opposite that of the initial configuration.

nonlinear interaction terms, which cannot be removed by adjusting the parameters. Moreover, the distribution in the final stage of the exact solutions is determined by the real parts of p_i and q_i and minor determinants of K, not by the velocities of dromions, which are the imaginary parts of p_i and q_i [11]. In our computations, however, the distribution in the final stage depends on the relative velocity. If we want to explain the results of our computations by the exact solutions, we must further impose relations between the components of H and the velocities of dromions. This makes the behaviors of the boundary conditions U and V more complicated, for a_{34} appears in (14). In fact, the intermediate states of the exact solutions vary remarkably by changing parameters such as the relative velocity of initial two dromions, the boundary conditions of potentials, and parameters in K [11,12]. Hence the behavior of $|A|^2$ during a collision is supposed to be different from the behavior found in our simulations.

Considering the findings above, we think that the oscillation of the intermediate pulse observed in our solutions, which universally occurs and is *independent* of the parameters, cannot exist in the exact solutions. Therefore we conclude that it is difficult to explain the distribution law in our simulation by using exact (2,2)-dromion solutions.

V. CONCLUDING REMARKS

In this paper, collisions of dromions are studied in detail numerically. We confirm that, in general, the initial two dromions will break into four pulses after collisions. This holds even if we make small changes on the mass ratio and the impact parameter. In addition, in the case of the collision of dromions with different masses, numerical results show that the initial two dromions exchange their masses through the collision.

The four pulses in the final stage are located around the cross point of the potentials, the area where the peaks of the mean flows intersect, and these intersections are the most attractive points in this system. We think that these four pulses would propagate stably if the ripples generated by the collision were sufficiently small. This is because if they sufficiently separate each other, then the interaction among pulses disappear; we can consider the state to be a special case of a superposition of four one-dromion solutions. The numerical results in this paper reveal the significant role of the mean flows in forming localized structures given by the DS1 equations. As we have already mentioned, these flows act as attractive potentials, so that the intersections of them maintain the localized structures of the main flow.

We have also observed the phenomenon that the intermediate pulse created as a result of a collision shows an oscillation while the potentials sufficiently overlap. The pulse has been observed to oscillate between two configurations, where it is flat along the line $x = \pm y$. One of the remarkable feature of the oscillation is that the period is independent of the relative velocity. Moreover, we find the distribution law that the state of the oscillation and the relative velocity determine the mass allocation of four pulses in the final stage.

As mentioned before, the DS1 equations are derived in a magnetized plasma [6]. Then a localized ion pulse will surely be formed and convey the plasma energy stably. The numerical results show that the energy is not scattered away even if they collide and the behavior of dromions obeys the distribution law.

At present, we think that it is difficult to explain the distribution law by using exact (2,2)-dromion solutions. Therefore, this law is considered to be essentially different. Further analyses on the mechanism of the oscillation and the distribution law and applications in real systems are problems to be studied in the future.

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APPENDIX

In this Appendix we discuss the problem of constructing the Hamiltonian of the DS1 equations for the dromion solution. For sufficiently localized pulses of A and Q, we can get (1) and (2) from variational problems

$$\frac{\delta H}{\delta A^*} = iA_t, \qquad \frac{\delta H}{\delta Q} = 0, \tag{A1}$$

where the Hamiltonian is

$$H = \int \Bigg(|A_x|^2 + |A_y|^2 + |A|^4 - (Q_x + Q_y)|A|^2 + \frac{Q_x Q_y}{2} \Bigg) dV. \tag{A2}$$

In the calculation of the variation in (A1), surface terms must vanish to obtain the DS1 equations. When we consider a dromion solution, the variable A decays exponentially in all directions. Then all of the surface terms disappear in the variation $\delta H/\delta A^*$ and we obtain (1). On the other hand, we cannot succeed in having (2) by $\delta H/\delta Q$ because the function Q is not localized. The variation of the last term in (A2) is

$$\delta \int \frac{1}{2} Q_x Q_y dV = \frac{1}{2} \int [Q_y \delta Q]_{x=-\infty}^{x=\infty} dy + \frac{1}{2} \int [Q_x \delta Q]_{y=-\infty}^{y=\infty} dx - \int Q_{xy} \delta Q dV. \quad \text{(A3)}$$

The first two terms of the right-hand side of (A3) must vanish to get a nontrivial Hamiltonian. This occurs only when the cross sections of Q at $x = +\infty$ and $y = +\infty$ coincide with those at $x = -\infty$ and $y = -\infty$, respectively. However, it is easily seen that these conditions are not satisfied in the case of the dromion solution, because it is not symmetrical with respect to the x and y axes. Thus we cannot construct the Hamiltonian in this case. Consequently, the Lyapunov analysis is not applicable to the examination of the stability of a dromion. This fact is quite natural from the physical point of view [7]. A dromion is located at a cross point of the peaks of two mean flows and we have to drive these mean flows from the boundaries of a system. For this reason, this system has an energy interaction with an external system through its boundaries. Therefore, it is apparent that this system is not a Hamiltonian system.

[1] M. Boiti, J. J.-P. Leon, L. Martina, and F. Pempinelli, Phys. Lett. A **132**, 432 (1988).

[2] M. J. Ablowitz and H. Segur, *Solitons and the Inverse Scattering Transform* (SIAM, Philadelphia, 1981), p. 322.

[3] A. S. Fokas and P. M. Santini, Phys. Rev. Lett. **63**, 1329 (1989).

[4] J. Hietarinta and R. Hirota, Phys. Lett. A **145**, 237 (1990).

[5] A. Davey and K. Stewartson, Proc. R. Soc. London Ser. A **338**, 101 (1974).

[6] K. Nishinari, K. Abe, and J. Satsuma, Phys. Plasmas **1**, 2559 (1994).

[7] K. Nishinari and T. Yajima, J. Phys. Soc. Jpn. **63**, 3538 (1994).

[8] E. A. Kuznetsov, A. M. Rubenchik, and V. E. Zakharov, Phys. Rep. **142** (3), 103 (1986).

[9] T. R. Taha and M. J. Ablowitz, J. Comput. Phys. **55**, 203 (1984).

[10] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes in C* (Cambridge University Press, London, 1992), p. 724.

[11] C. R. Gilson and J. J. C. Nimmo, Proc. R. Soc. London Ser. A **435**, 339 (1991).

[12] P. M. Santini, Physica D **41**, 26 (1990).